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ON THERMODYNAMIC OF DARK ENERGY

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The foundations of dynamical cosmology in homogeneous and isotropic space with the Friedmann-Lemeter-Robertson-Walker (FLRW) metric $d\tau^2 = c^2 dt^2 - a^2(t) dl^2$ are:

a) Consequence from Einstein's equations in the form of the Friedman equation

$$H^2(t) = \Gamma u - \frac{\kappa c^2}{a^2(t)} \quad ,$$

and covariant conservation conditions for the energy-momentum tensor

$$\frac{du}{P+u} = -3\frac{da}{a} = -3H(t)dt \quad ,$$

where: $H(t) \equiv \frac{\dot{a}(t)}{a(t)}$, $\quad \Gamma = \frac{8\pi G}{3}$

Γ — Newton's constant

$H(t)$ — Hubble's parameter

$a(t)$ — scale factor of the given metric

$\dot{a}(t)$ — time derivative

t — synchronous world time

c — speed of light

dl^2 — dimensionless element of length of three dimensional space constant curvature with singe $\kappa = 0, \pm 1$

and **b)** The equation of state of a cosmological medium as the dependence between its pressure $P = P(u)$ and internal energy U :

$$P = \lambda u$$

$$\frac{PV}{\lambda} = U(T,V,N), \quad u(T,\overline{\upsilon}) = \frac{U}{V} \ ,$$

$$\sigma(T,\overline{\upsilon}) = \frac{S}{V} \ , \quad \text{also} \quad \overline{\upsilon} = \frac{V}{N} \ ,$$

$$\bar{\varepsilon}(T,\overline{\upsilon}) = \frac{U}{N} \ , \qquad \bar{s}(T,\overline{\upsilon}) = \frac{S}{N} \ .$$

$\lambda(V) = \varpi(\overline{\upsilon})$ — the **Grüneisen parameter**.

The co-moving volume: $V \mapsto V_a = [a(t)\Delta l]^3$

For any volume at internal energy $U(S,V,N) = Vu(\bar{s},\bar{\upsilon})$:

$$P = -\left(\frac{\partial U}{\partial V}\right)_{S,N} = -\left(\frac{\partial \bar{\varepsilon}}{\partial \bar{\upsilon}}\right)_{\bar{s}} = -\left(\frac{\partial(\bar{\upsilon}u)}{\partial \bar{\upsilon}}\right)_{\bar{s}}$$

$$\lambda = const$$

$$u(\bar{s},\bar{\upsilon}) = \overline{B}_\lambda(\bar{s})\bar{\upsilon}^{-\lambda-1} = \widetilde{B}_\lambda(S,N)V^{-\lambda-1} \mapsto B_\lambda\left[a^3(t)(\Delta l)^3\right]^{-\lambda-1}$$

$\widetilde{B}_\lambda(S,N) = N^{\lambda+1}\overline{B}_\lambda(\bar{s})$, namely, in the form of

$$B_\lambda \equiv \widetilde{B}_\lambda(S_a,\mathcal{N}_a) = \mathcal{N}_a^{\lambda+1}\overline{B}_\lambda(\bar{s})$$

$$\lambda = 1/3;\ 0; -1$$

Ultrarelativistic medium or radiation	Nonrelativistic cold medium (dark and/or baryonic "dust")	Vacuum

The interval for the lambda $-1 < \lambda < 0$ corresponds to the so-called quintessence as some "pre-vacuum" medium where lambda is a function of time.

For example, in dynamic scalar field models, the quintessence is given by a homogeneous time-dependent scalar field $\varphi(t)$ with a potential $\mathrm{V}(\varphi)$

$$\lambda(V_a) \equiv \varpi(\overline{\upsilon}(t)) \mapsto w(\varphi(t)) = \frac{P_\varphi}{u_\varphi} = \frac{\dot{\varphi}^2/2 - \mathrm{V}(\varphi)}{\dot{\varphi}^2/2 + \mathrm{V}(\varphi)}$$

Thermodynamic functions of the medium $P=\lambda u$

$$\left(\frac{\partial\bar{\varepsilon}(T,\bar{\upsilon})}{\partial\bar{\upsilon}}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_{\bar{\upsilon}} - P \quad , \qquad \left(\frac{\partial U(T,V,N)}{\partial V}\right)_{T,N} = T\left(\frac{\partial P}{\partial T}\right)_{V,N} - P$$

$$\bar{\varepsilon}(T,\varsigma)=\left(T\frac{\partial}{\partial T}+\varsigma\frac{\partial}{\partial\varsigma}\right)\bar{\varepsilon}(T,\varsigma) \quad , \qquad U(T,\xi)=\left(T\frac{\partial}{\partial T}+\xi\frac{\partial}{\partial\xi}\right)U(T,\xi)$$

$$\frac{d\ln\varsigma(\bar{\upsilon})}{d\ln\bar{\upsilon}}=-\varpi(\bar{\upsilon}) \quad , \qquad \varsigma(\bar{\upsilon})=\varsigma_0\exp\left\{-\int_{\bar{\upsilon}_0}^{\bar{\upsilon}}\frac{d\eta}{\eta}\varpi(\eta)\right\} \quad , \qquad \varsigma_0=\varsigma(\bar{\upsilon}_0)$$

$$\frac{d\ln\xi(V)}{d\ln V}=-\lambda(V) \quad , \qquad \xi(V)=\xi_0\exp\left\{-\int_{V_0}^{V}\frac{dv}{v}\lambda(v)\right\} \quad , \qquad \xi_0=\xi(V_0)$$

Solutions in the form of $z \equiv T/\xi(V)$ or $q \equiv T/\varsigma(\bar{\upsilon})$:

$$\bar{\varepsilon}(T,\varsigma) = \varsigma\chi\left(\frac{T}{\varsigma}\right) \text{ , or } U(T,\xi) = \xi f\left(\frac{T}{\xi}\right) ,$$

$$\text{and} \quad P(T,\bar{\upsilon}) = \frac{\varpi(\bar{\upsilon})}{\bar{\upsilon}}\varsigma\chi\left(\frac{T}{\varsigma}\right) = \frac{\lambda(V)}{V}\xi f\left(\frac{T}{\xi}\right) ,$$

$$u(\bar{s},\bar{\upsilon}) = \bar{B}_{\varpi}(\bar{s})\frac{\varsigma(\bar{\upsilon})}{\bar{\upsilon}} = \tilde{B}_{\varpi}(S,N)\frac{\xi(V)}{V} \mapsto B_{\varpi}\frac{\xi(V_a)}{V_a} ,$$

$$\bar{B}_{\varpi}(\bar{s}) = \chi(q) , \quad \tilde{B}_{\varpi}(S,N) = f(z;N) \text{ and FI } B_{\varpi} \equiv \tilde{B}_{\varpi}(S_a,\mathcal{N}_a)$$

$$C_{V,N} = \left(\frac{\partial U}{\partial T}\right)_{V,N} = f'(z) = N\overline{C}_{\overline{\upsilon}} \, , \quad \overline{C}_{\overline{\upsilon}} = \left(\frac{\partial \varepsilon}{\partial T}\right)_{\overline{\upsilon}} = \chi'(q) \, ,$$

$$S(z)\big|_N = \int_{[0]}^{z} dx \frac{f'(x)}{x} \quad , \quad \text{and} \quad \bar{s}(q) = \int_{[0]}^{q} dy \frac{\chi'(y)}{y}$$

$$\mu(T,\overline{\upsilon}) = [1+\varpi(\overline{\upsilon})]\varsigma(\overline{\upsilon})\chi(q) - T\bar{s}(q) \, , \qquad \phi(q,\overline{\upsilon}) \equiv \frac{\mu(T,\overline{\upsilon})}{T} = [1+\varpi(\overline{\upsilon})]\frac{\chi(q)}{q} - \bar{s}(q)$$

$$\frac{\chi(q)}{q} = \frac{1}{\varpi'(\overline{\upsilon})}\left(\frac{\partial \phi(q,\overline{\upsilon})}{\partial \overline{\upsilon}}\right)_q \, ,$$

for $\lambda(V) = \varpi(\overline{\upsilon}) \neq const$

$$\frac{\chi(q)}{q} = q^{1/\lambda}\left[A_\lambda + \int_{[0]}^{q} dy \frac{\phi'(y)}{\lambda y^{1/\lambda}}\right],$$

for $\lambda = const \neq 0$, $\phi(q,\overline{\upsilon}) \mapsto \phi(q)$

The positive constant $A_\lambda > 0$ defines here the solution

$$\chi_{(\lambda)}(q) = A_\lambda q^{1+1/\lambda} \, , \quad \mu = 0 \, ,$$

$$\phi(q) = const = \phi_0$$

$$S(z) = const \quad , \qquad \bar{s}(q) = const$$

$$q \Rightarrow \beta(\bar{s}) = \overline{B}'_{\varpi}(\bar{s}) \quad , \qquad \chi(q) \equiv \frac{\bar{\varepsilon}(T, \bar{\upsilon})}{\varsigma(\bar{\upsilon})} \Rightarrow \overline{B}_{\varpi}(\bar{s}(\beta)) = \chi(\beta) \quad ,$$

$$z \Rightarrow b(S) \quad , \qquad f(z; N) \equiv \frac{U(T, V, N)}{\xi(V)} \Rightarrow \tilde{B}_{\varpi}(S(b), N) = f(b) \; .$$

Thermodynamics of quintessence

$$\overline{C}_{\overline{\upsilon}} > 0 \quad , \; -\overline{\upsilon}\left(\frac{\partial P}{\partial \overline{\upsilon}}\right)_{\overline{s}} > -\overline{\upsilon}\left(\frac{\partial P}{\partial \overline{\upsilon}}\right)_{T} \geq 0 \quad , \text{ or}$$

$$\chi'(q) > 0 \, , \quad 1 + \frac{d}{d\overline{\upsilon}}\left(\frac{\overline{\upsilon}}{\varpi(\overline{\upsilon})}\right) \geq \frac{d \ln \chi(q)}{d \ln q} > 0$$

There exists a constant $\alpha > 0$ which limits the degree of growth of the increasing, differentiable in given interval $0 \leq q < \infty$ function of q , $\chi(q) \leq \chi_{[\alpha]}(q) \equiv A_{[\alpha]} q^{\alpha}$, by the degree of growth of the function of $\overline{\upsilon}$: at $\lambda(V) = \varpi(\overline{\upsilon}) \neq const$ in the form

$$1 + \frac{d}{d\overline{\upsilon}}\left(\frac{\overline{\upsilon}}{\varpi(\overline{\upsilon})}\right) \geq \alpha = \max_{q}\left[\frac{d \ln \chi(q)}{d \ln q}\right] > 0$$

When $\lambda = const$, it leads to the constraint: $1+\frac{1}{\lambda} \geq \alpha > 0$

So at least $\lambda \geq 0$, or $\lambda < -1$ but not $-1 < \lambda < 0$

The conditions on the non-constant function $\varpi(\bar{\upsilon}) \neq const$, $0 < \bar{\upsilon} < \infty$:

$$\min_{\bar{\upsilon}} \left[\frac{d}{d\bar{\upsilon}} \left(\frac{\bar{\upsilon}}{\varpi(\bar{\upsilon})} \right) \right] \geq \alpha - 1 \quad , \quad -1 < \varpi(\bar{\upsilon}) < 0$$

For the function $\Xi(\bar{\upsilon}) = -\frac{\bar{\upsilon}}{\varpi(\bar{\upsilon})}$: $\max_{\bar{\upsilon}} \left[\Xi'(\bar{\upsilon}) \right] \leq 1 - \alpha \, , \; \bar{\upsilon} < \Xi(\bar{\upsilon}) < \infty$

Which contradict each other since $\alpha > 0$

Conclusion

Thus quintessence in the form of a one-component medium with the equation of state $P = \lambda u$ turns out to be impossible as a macroscopically stable thermodynamically equilibrium medium for any function $-1 < \lambda(V) = \varpi(\bar{\upsilon}) < 0$

About dark energy

$$\chi(q) = B_\lambda \left[\frac{\bar{\upsilon}}{a^3}\right]^{\lambda+1} \equiv B_\lambda \left[\frac{q}{Z_\lambda(a)}\right]^{1+1/\lambda},$$

$$f(z) = B_\lambda \left[\frac{V}{a^3}\right]^{\lambda+1} \equiv B_\lambda \left[\frac{z}{Z_\lambda(a)}\right]^{1+1/\lambda},$$

$$Z_\lambda(a) = T[a(t)]^{3\lambda}$$

$$-P = u = \chi = A = const\,,$$

with $\Delta S = C_V = C_P = C_S \equiv 0$,

$U = AV$, at $T = 0$,

$\mu = 0$, $S_0 \neq 0$

$$T\sigma + \mu n = P + u$$

where $n = 1/\bar{\upsilon}$ ⇒ $T = 0$

Thank you for your attention!

$$\lambda = const$$

$$\xi = V^{-\lambda}, \quad \varsigma = \overline{\upsilon}^{-\lambda},$$

$$z = TV^{\lambda},$$

$$q = T\overline{\upsilon}^{\lambda} = zN^{-\lambda}$$

Equations of state: $\bar{\varepsilon}(T,\overline{\upsilon}) = \overline{\upsilon}^{-\lambda}\chi\left(T\overline{\upsilon}^{\lambda}\right)$,

$$U(T,V,N) = V^{-\lambda} f\left(TV^{\lambda};N\right),$$

$$P = \frac{\lambda}{V^{\lambda+1}} f\left(TV^{\lambda};N\right) = \frac{\lambda}{\overline{\upsilon}^{\lambda+1}}\chi\left(T\overline{\upsilon}^{\lambda}\right),$$

$$u\left(T,\overline{\upsilon}\right) = \frac{\bar{\varepsilon}(T,\overline{\upsilon})}{\overline{\upsilon}} = \frac{\chi(q)}{\overline{\upsilon}^{\lambda+1}}, \quad \sigma\left(T,\overline{\upsilon}\right) = \frac{\bar{s}\left(T\overline{\upsilon}^{\lambda}\right)}{\overline{\upsilon}},$$

Adiabatic process equations:

$$q \equiv T\overline{\upsilon}^{\lambda} \Rightarrow \beta, \quad \bar{B}_{\lambda}\left(\bar{s}\right) = \chi\left(\beta\right), \quad P\overline{\upsilon}^{\lambda+1} \Rightarrow \lambda\chi\left(\beta\right), \quad \frac{\mu}{T} \Rightarrow \phi\left(\beta\right)$$

$$\chi_{[0]}(q) = \mathrm{A}_{[0]},$$

$$\chi_{[1]}(q) = \overline{\mathrm{A}}q,$$

$$\bar{\varepsilon}_{[0]}(\overline{\upsilon}) = \mathrm{A}_{[0]}\varsigma(\overline{\upsilon}), \quad U_{[0]} = N\bar{\varepsilon}_{[0]}$$

$$\bar{\varepsilon}_{[1]}(T) = \overline{\mathrm{A}}T, \quad U_{[1]} = N\bar{\varepsilon}_{[1]}$$

The general solution is a superposition of $\chi(q) = \chi_{[0]}(q) + \chi_{[1]}(q) + \chi_{(\lambda)}(q) + \ldots$

$$\chi_{(\lambda)}(q) = A_\lambda q^{1+1/\lambda}, \quad f_{(\lambda)}(z) = A_\lambda z^{1+1/\lambda}, \quad U_{(\lambda)}(T,V) = A_\lambda V T^{1+1/\lambda},$$

$$S_{(\lambda)}(z) = (\lambda+1)A_\lambda V T^{1/\lambda}, \quad P_{(\lambda)}(T) = \lambda u_{(\lambda)}(T) = \lambda A_\lambda T^{1+1/\lambda},$$

$$C_{V(\lambda)} = \left(1 + \frac{1}{\lambda}\right) A_\lambda V T^{1/\lambda}, \quad \mu_{(\lambda)} = 0$$

,