

**Baikal School of Fundamental Physics  
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# **ON THERMODYNAMIC OF DARK ENERGY**

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The foundations of dynamical cosmology in homogeneous and isotropic space with the Friedmann-Lemeter-Robertson-Walker (FLRW) metric

$$d\tau^2 = c^2 dt^2 - a^2(t) dl^2 \text{ are:}$$

a) Consequence from Einstein's equations in the form of the Friedman equation

and covariant conservation conditions for the energy-momentum tensor

$\Gamma$  — Newton's constant

$H(t)$  — Hubble's parameter

$a(t)$  — scale factor of the given metric

$\dot{a}(t)$  — time derivative

$t$  — synchronous world time

$c$  — speed of light

$dl^2$  — dimensionless element of length of three-dimensional space constant curvature with singe  $\kappa = 0, \pm 1$

$$H^2(t) = \Gamma u - \frac{\kappa c^2}{a^2(t)} ,$$

$$\frac{du}{P+u} = -3 \frac{da}{a} = -3H(t)dt ,$$

$$\text{where: } H(t) \equiv \frac{\dot{a}(t)}{a(t)}, \quad \Gamma = \frac{8\pi G}{3}$$

and **b)** The equation of state of a cosmological medium as the dependence between its pressure  $P = P(u)$  and internal energy  $U$  :

$$P = \lambda u$$

$$\frac{PV}{\lambda} = U(T, V, N), \quad u(T, \bar{v}) = \frac{U}{V},$$

$$\sigma(T, \bar{v}) = \frac{S}{V}, \quad \text{also} \quad \bar{v} = \frac{V}{N},$$

$$\bar{\varepsilon}(T, \bar{v}) = \frac{U}{N}, \quad \bar{s}(T, \bar{v}) = \frac{S}{N}.$$

$\lambda(V) = \varpi(\bar{v})$  – the **Grüneisen parameter**.

The co-moving volume:  $V \mapsto V_a = [a(t)\Delta l]^3$

For any volume at internal energy  $U(S, V, N) = Vu(\bar{s}, \bar{v})$ :

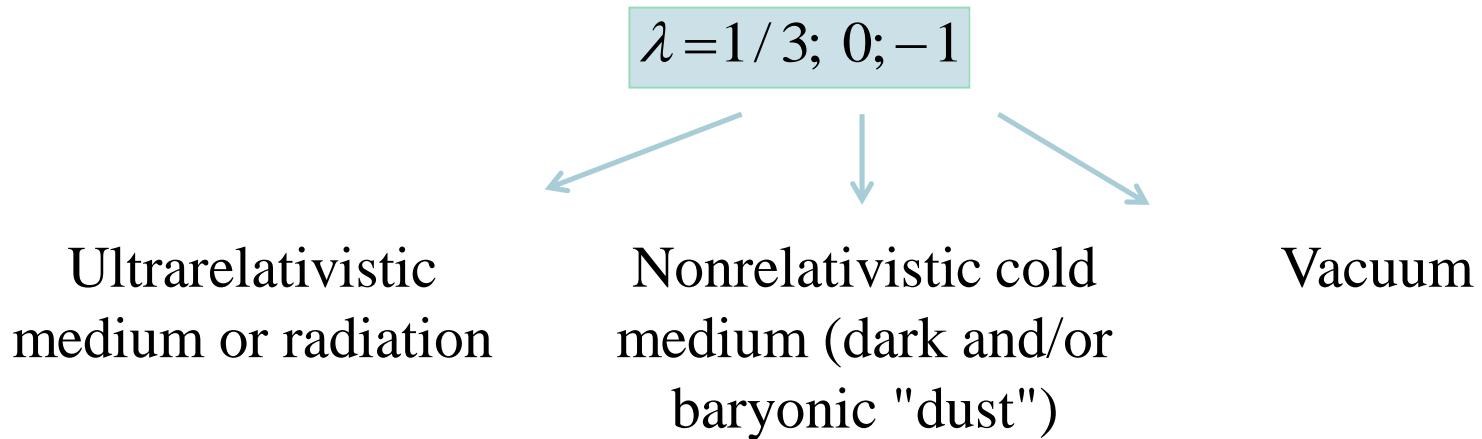
$$P = -\left(\frac{\partial U}{\partial V}\right)_{S,N} = -\left(\frac{\partial \bar{e}}{\partial \bar{v}}\right)_{\bar{s}} = -\left(\frac{\partial (\bar{v}u)}{\partial \bar{v}}\right)_{\bar{s}}$$

$$\lambda = \text{const}$$

$$u(\bar{s}, \bar{v}) = \bar{B}_\lambda(\bar{s}) \bar{v}^{-\lambda-1} = \tilde{B}_\lambda(S, N) V^{-\lambda-1} \mapsto B_\lambda [a^3(t)(\Delta l)^3]^{-\lambda-1}$$

$\tilde{B}_\lambda(S, N) = N^{\lambda+1} \bar{B}_\lambda(\bar{s})$ , namely, in the form of

$$B_\lambda \equiv \tilde{B}_\lambda(S_a, \mathcal{N}_a) = \mathcal{N}_a^{\lambda+1} \bar{B}_\lambda(\bar{s})$$



The interval for the lambda  $-1 < \lambda < 0$  corresponds to the so-called quintessence as some "pre-vacuum" medium where lambda is a function of time.

For example, in dynamic scalar field models, the quintessence is given by a homogeneous time-dependent scalar field  $\phi(t)$  with a potential  $V(\phi)$

$$\lambda(V_a) \equiv \varpi(\bar{v}(t)) \mapsto w(\phi(t)) = \frac{P_\phi}{u_\phi} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}$$

# Thermodynamic functions of the medium $P = \lambda u$

$$\left( \frac{\partial \bar{\varepsilon}(T, \bar{v})}{\partial \bar{v}} \right)_T = T \left( \frac{\partial P}{\partial T} \right)_{\bar{v}} - P \quad , \quad \left( \frac{\partial U(T, V, N)}{\partial V} \right)_{T, N} = T \left( \frac{\partial P}{\partial T} \right)_{V, N} - P$$

$$\bar{\varepsilon}(T, \xi) = \left( T \frac{\partial}{\partial T} + \xi \frac{\partial}{\partial \xi} \right) \bar{\varepsilon}(T, \xi) \quad , \quad U(T, \xi) = \left( T \frac{\partial}{\partial T} + \xi \frac{\partial}{\partial \xi} \right) U(T, \xi)$$

$$\frac{d \ln \xi(\bar{v})}{d \ln \bar{v}} = -\varpi(\bar{v}) \quad , \quad \xi(\bar{v}) = \xi_0 \exp \left\{ - \int_{\bar{v}_0}^{\bar{v}} \frac{d\eta}{\eta} \varpi(\eta) \right\} \quad , \quad \xi_0 = \xi(\bar{v}_0)$$

$$\frac{d \ln \xi(V)}{d \ln V} = -\lambda(V) \quad , \quad \xi(V) = \xi_0 \exp \left\{ - \int_{V_0}^V \frac{dv}{v} \lambda(v) \right\} \quad , \quad \xi_0 = \xi(V_0)$$

Solutions in the form of  $z \equiv T/\xi(V)$  or  $q \equiv T/\xi(\bar{v})$  :

$$\bar{\varepsilon}(T, \xi) = \xi \chi\left(\frac{T}{\xi}\right), \text{ or } U(T, \xi) = \xi f\left(\frac{T}{\xi}\right),$$

$$\text{and } P(T, \bar{v}) = \frac{\varpi(\bar{v})}{\bar{v}} \xi \chi\left(\frac{T}{\xi}\right) = \frac{\lambda(V)}{V} \xi f\left(\frac{T}{\xi}\right),$$

$$u(\bar{s}, \bar{v}) = \bar{B}_\varpi(\bar{s}) \frac{\xi(\bar{v})}{\bar{v}} = \tilde{B}_\varpi(S, N) \frac{\xi(V)}{V} \mapsto B_\varpi \frac{\xi(V_a)}{V_a},$$

$$\bar{B}_\varpi(\bar{s}) = \chi(q), \quad \tilde{B}_\varpi(S, N) = f(z; N) \quad \text{and FI} \quad B_\varpi \equiv \tilde{B}_\varpi(S_a, \mathcal{N}_a)$$

$$C_{V,N} = \left( \frac{\partial U}{\partial T} \right)_{V,N} = f'(z) = N \bar{C}_{\bar{v}} , \quad \bar{C}_{\bar{v}} = \left( \frac{\partial \bar{\epsilon}}{\partial T} \right)_{\bar{v}} = \chi'(q) ,$$

$$S(z) \Big|_N = \int_0^z dx \frac{f'(x)}{x} \quad , \text{ and} \quad \bar{s}(q) = \int_0^q dy \frac{\chi'(y)}{y}$$

$$\mu(T, \bar{v}) = [1 + \varpi(\bar{v})] \varsigma(\bar{v}) \chi(q) - T \bar{s}(q), \quad \phi(q, \bar{v}) \equiv \frac{\mu(T, \bar{v})}{T} = [1 + \varpi(\bar{v})] \frac{\chi(q)}{q} - \bar{s}(q)$$

$$\frac{\chi(q)}{q} = \frac{1}{\varpi'(\bar{v})} \left( \frac{\partial \phi(q, \bar{v})}{\partial \bar{v}} \right)_q ,$$

for  $\lambda(V) = \varpi(\bar{v}) \neq \text{const}$

$$\frac{\chi(q)}{q} = q^{1/\lambda} \left[ A_\lambda + \int_0^q dy \frac{\phi'(y)}{\lambda y^{1/\lambda}} \right],$$

The positive constant  $A_\lambda > 0$  defines here the solution

$$\chi_{(\lambda)}(q) = A_\lambda q^{1+1/\lambda}, \quad \mu = 0 ,$$

$$\phi(q) = \text{const} = \phi_0$$

for  $\lambda = \text{const} \neq 0$ ,  $\phi(q, \bar{v}) \mapsto \phi(q)$

$$S(z) = \text{const} , \quad \bar{s}(q) = \text{const}$$

$$q \Rightarrow \beta(\bar{s}) = \bar{B}'_{\varpi}(\bar{s}) , \quad \chi(q) \equiv \frac{\bar{\varepsilon}(T, \bar{v})}{\varsigma(\bar{v})} \Rightarrow \bar{B}_{\varpi}(\bar{s}(\beta)) = \chi(\beta) ,$$

$$z \Rightarrow b(S) , \quad f(z; N) \equiv \frac{U(T, V, N)}{\xi(V)} \Rightarrow \tilde{B}_{\varpi}(S(b), N) = f(b) .$$

# Thermodynamics of quintessence

$$\bar{C}_v > 0 , -\bar{v} \left( \frac{\partial P}{\partial \bar{v}} \right)_{\bar{s}} > -\bar{v} \left( \frac{\partial P}{\partial \bar{v}} \right)_T \geq 0 , \text{ or}$$

$$\chi'(q) > 0 , 1 + \frac{d}{d\bar{v}} \left( \frac{\bar{v}}{\varpi(\bar{v})} \right) \geq \frac{d \ln \chi(q)}{d \ln q} > 0$$



There exists a constant  $\alpha > 0$  which limits the degree of growth of the increasing, differentiable in given interval  $0 \leq q < \infty$  function of  $q$  ,  
 $\chi(q) \leq \chi_{[\alpha]}(q) \equiv A_{[\alpha]} q^\alpha$  , by the degree of growth of the function of  $\bar{v}$  :  
at  $\lambda(V) = \varpi(\bar{v}) \neq \text{const}$  in the form

$$1 + \frac{d}{d\bar{v}} \left( \frac{\bar{v}}{\varpi(\bar{v})} \right) \geq \alpha = \max_q \left[ \frac{d \ln \chi(q)}{d \ln q} \right] > 0$$



When  $\lambda = \text{const}$ , it leads to the constraint:

$$1 + \frac{1}{\lambda} \geq \alpha > 0$$

So at least

$$\lambda \geq 0, \text{ or } \lambda < -1$$

but not

$$-1 < \lambda < 0$$

The conditions on the non-constant function  $\varpi(\bar{v}) \neq \text{const}$ ,  $0 < \bar{v} < \infty$ :

$$\min_{\bar{v}} \left[ \frac{d}{d\bar{v}} \left( \frac{\bar{v}}{\varpi(\bar{v})} \right) \right] \geq \alpha - 1, \quad -1 < \varpi(\bar{v}) < 0$$

For the function  $\Xi(\bar{v}) = -\frac{\bar{v}}{\varpi(\bar{v})}$ :

$$\max_{\bar{v}} [\Xi'(\bar{v})] \leq 1 - \alpha, \quad \bar{v} < \Xi(\bar{v}) < \infty$$

Which contradict each other since  $\alpha > 0$

# Conclusion

Thus quintessence in the form of a one-component medium with the equation of state  $P = \lambda u$  turns out to be impossible as a macroscopically stable thermodynamically equilibrium medium for any function  
 $-1 < \lambda(V) = \varpi(\bar{v}) < 0$

# About dark energy

$$\chi(q) = B_\lambda \left[ \frac{\bar{v}}{a^3} \right]^{\lambda+1} \equiv B_\lambda \left[ \frac{q}{Z_\lambda(a)} \right]^{1+1/\lambda},$$

$$f(z) = B_\lambda \left[ \frac{V}{a^3} \right]^{\lambda+1} \equiv B_\lambda \left[ \frac{z}{Z_\lambda(a)} \right]^{1+1/\lambda},$$

$$Z_\lambda(a) = T[a(t)]^{3\lambda}$$

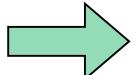
$$-P = u = \chi = A = \text{const},$$

$$\text{with } \Delta S = C_V = C_P = C_S \equiv 0,$$

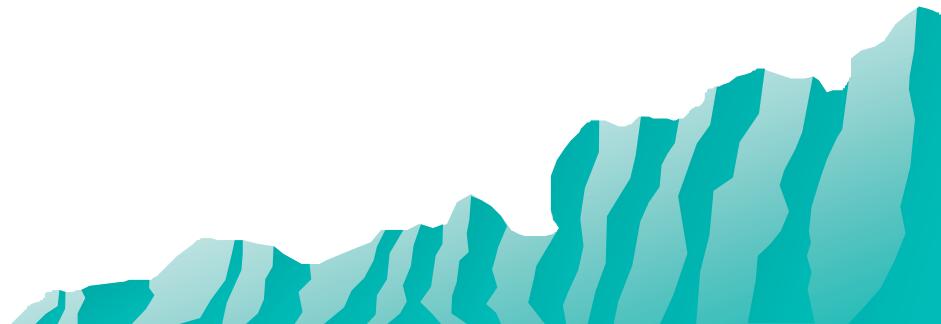
$$U = AV, \text{ at } T = 0,$$

$$\mu = 0, \quad S_0 \neq 0$$

$$T\sigma + \mu n = P + u$$

where  $n = 1/\bar{v}$    $T = 0$

Thank you for your attention!



$$\lambda = \text{const}$$

$$\xi = V^{-\lambda}, \quad \varsigma = \bar{v}^{-\lambda},$$

$$z = TV^\lambda,$$

$$q = T\bar{v}^\lambda = zN^{-\lambda}$$

$$\text{Equations of state: } \bar{\varepsilon}(T, \bar{v}) = \bar{v}^{-\lambda} \chi(T\bar{v}^\lambda),$$

$$U(T, V, N) = V^{-\lambda} f(TV^\lambda; N),$$

$$P = \frac{\lambda}{V^{\lambda+1}} f(TV^\lambda; N) = \frac{\lambda}{\bar{v}^{\lambda+1}} \chi(T\bar{v}^\lambda),$$

$$u(T, \bar{v}) = \frac{\bar{\varepsilon}(T, \bar{v})}{\bar{v}} = \frac{\chi(q)}{\bar{v}^{\lambda+1}}, \quad \sigma(T, \bar{v}) = \frac{\bar{s}(T\bar{v}^\lambda)}{\bar{v}},$$

Adiabatic process equations:

$$q \equiv T\bar{v}^\lambda \Rightarrow \beta, \quad \bar{B}_\lambda(\bar{s}) = \chi(\beta), \quad P\bar{v}^{\lambda+1} \Rightarrow \lambda\chi(\beta), \quad \frac{\mu}{T} \Rightarrow \phi(\beta)$$

$$\chi_{[0]}(q) = A_{[0]} ,$$

$$\chi_{[1]}(q) = \bar{A}q ,$$

$$\bar{\varepsilon}_{[0]}(\bar{v}) = A_{[0]} \zeta(\bar{v}) , \quad U_{[0]} = N \bar{\varepsilon}_{[0]} \quad \bar{\varepsilon}_{[1]}(T) = \bar{A}T , \quad U_{[1]} = N \bar{\varepsilon}_{[1]}$$

The general solution is a superposition of  $\chi(q) = \chi_{[0]}(q) + \chi_{[1]}(q) + \chi_{(\lambda)}(q) + \dots$

$$\chi_{(\lambda)}(q) = A_\lambda q^{1+1/\lambda} , \quad f_{(\lambda)}(z) = A_\lambda z^{1+1/\lambda} , \quad U_{(\lambda)}(T, V) = A_\lambda V T^{1+1/\lambda} ,$$



$$S_{(\lambda)}(z) = (\lambda + 1) A_\lambda V T^{1/\lambda} , \quad P_{(\lambda)}(T) = \lambda u_{(\lambda)}(T) = \lambda A_\lambda T^{1+1/\lambda} ,$$

$$C_{V(\lambda)} = \left(1 + \frac{1}{\lambda}\right) A_\lambda V T^{1/\lambda} , \quad \mu_{(\lambda)} = 0$$

,