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К ВОПРОСУ О ДИНАМИКЕ РАДИАЦИОННЫХ ПОЯСОВ ЗЕМЛИ

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ON THE EARTH'S RADIATION BELT DYNAMICS

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Аннотация. Резонансное взаимодействие электронов со свистовыми волнами значительно влияет на динамику плазменных систем за счет рассеяния по питч-углу и ускорения электронов. При достаточно широком спектре волн или сильной неоднородности внешнего магнитного поля динамика подобных систем подчиняется квазилинейной теории. Однако в околоземном плазменном пространстве регулярно наблюдаются высокоинтенсивные свистовые волны, которые не могут быть описаны данным приближением вследствие наличия нелинейных резонансных эффектов. В этой работе рассматривается применение диффузионной теории для таких интенсивных волн с учетом волновых модуляций, которые значительно подавляют эффекты, связанные с нелинейным резонансным взаимодействием.

Ключевые слова: резонансное взаимодействие волн и электронов, свистовые волны, теория диффузии, квазилинейная теория, нелинейное резонансное взаимодействие.

Abstract. Resonant electron interaction with whistler-mode waves significantly affects the dynamics of plasma systems through pitch-angle scattering and particle acceleration. With a sufficiently wide wave spectrum or a strong inhomogeneity of the background magnetic field, the dynamics of such systems obey the quasi-linear theory. But many of the observed high-intensity waves cannot be described by this approximation due to the presence of nonlinear effects. In this paper, we consider the application of diffusion theory for waves of this type, taking into account wave modulations, which significantly suppresses the effects associated with nonlinear interaction.

Keywords: resonant wave-particle interaction, whistler-mode waves, diffusion theory, quasi-linear theory, nonlinear resonant interaction.

INTRODUCTION

Resonant electron interaction with whistler-mode waves is one of the main drivers of electron pitch-angle scattering and acceleration in various space plasma systems, e.g. solar wind [Tong et al., 2019] shock waves [Hull et al., 2012; Wilson et al., 2013; Oka et al., 2017; Page et al., 2021], planetary radiation belts [Menietti et al., 2002], and magnetic reconnection regions [Deng, Matsumoto, 2001]. The basic theoretical framework for description of such interaction is the quasi-linear model [Vedenov et al., 1962; Drummond, Pines, 1962; Andronov, Trakhtengerts, 1964; Kennel, Engelmann, 1966] that is based on assumption of weak perturbation of particle dynamics by each single resonance. This assumption reduces the Vlasov equation to the Fokker–Plank diffusion equation [Vedenov et al., 1962; Drummond, Pines, 1962] where the main characteristics of wave-particle resonant interactions are diffusion rates. The requirement of a weak perturbation of particle trajectories for a single resonance is equivalent to the requirement that such interaction should not last for a long time, and there are several mechanisms responsible for particle escape from the resonance.

The original quasi-linear diffusion model assumes the broad spectrum of waves resonating with charged particles [Vedenov et al., 1962; Drummond, Pines, 1962], when the resonance width in velocity space Δv_R equals to the difference of resonance v_R velocity and wave group velocity $v_g = \partial\omega/\partial k$ (where ω and k are wave frequency and wave number). Thus, change of the resonant particle velocity on $\Delta v_R \sim |v_R - v_g|\Delta k / k$ will remove particle from the resonance. As small factor $\Delta k/k$ is determined by the wave spectrum width in wavenumber space, Δk. This mechanism determines the shortness of individual resonance and justifies applicability of the diffusion approximation for modeling the dynamics of charged particle ensemble [Karpman, 1974; Le Queau, Roux, 1987; Shapiro, Sagdeev, 1997]. This description works well for low-amplitude whistler-mode waves resonating with electrons in homogeneous systems (without spatial gradients of the background plasma and magnetic field), e.g. in the solar wind?

An assumption about background magnetic field homogeneity, however, does no work for many space plasma systems. Resonant electron scattering by whistler-mode waves are often observed in magnetic field traps, regions with spatially localized minimum of magnetic field magnitude, where charged particles can be trapped and bouncing. Important examples of such traps are radiation belt dipole field [Lyons, Williams, 1984; Schulz, Lanzerotti, 1974] and magnetic holes generated by compressional perturbations on the bow shock [Oka et al., 2019]. Bouncing within magnetic traps, electrons periodically resonate with whistler-mode waves and resonance width for such interactions is determined by the spatial (field-aligned) gradient of the resonant velocity $\Delta v_R \sim |\partial v_R/\partial s|/k$ [Trakhtengerts, Rycroft, 2008]. If Δv_R is finite, the quasi-linear diffusion model works even for monochromatic waves ($\Delta k \to 0$) resonating with electrons in magnetic traps [Albert, 2001;

Albert, 2010]. Thus, the only condition required for application of quasi-linear diffusion mode is the smallest of wave intensity: the mirror force due to background magnetic field gradient should be stronger than the Lorentz force of wave field [Karpman, 1974].

The small wave intensity approximation, however, is often violated for whistler-mode waves observed in highly unstable plasma of shock waves [Zhang et al., 1999; Artemyev et al., 2022] and plasma injections [Zhang et al., 2018b; Zhang et al., 2019]. Such intense waves may resonate with electrons in nonlinear regime, including effects of phase trapping and phase bunching [Nunn, 1971; Nunn, 1974; Karpman et al., 1974; Inan et al., 1978]. Although phase bunching is the strongly nonlinear effect [Albert, 1993; Bortnik et al., 2008], due to smallness of electron energy and pitch-angle changes in a single resonant phase bunching, it can be incorporated as a drift term into the Fokker Plank equation [Artemyev et al., 2014; Allanson et al., 2021; Gan et al., 2020]. Changes of electron energy and pitch-angle due to the phase trapping are comparable to the initial energies/pitch-angles [Omura et al., 2007; Summers, Omura, 2007], and thus this effect cannot be described by differential operators in the Fokker–Plank equation. Several approaches with different integral operators describing the phase trapping contribution to the electron flux dynamics have been proposed [Omura et al., 2015; Artemyev et al., 2018; Vainchtein et al., 2018; Hsieh et al., 2020; Allanson et al., 2022], but evaluation of such operators is computationally expansive and significantly change the Fokker–Plank equation. Thus, it’s important and practically useful to propose an approach for incorporation of nonlinear effects without significant altering models based on the Fokker–Plank equation.

The principal possibility for such approach has been proposed in [Solovev, Shkliar, 1986]: the total contribution of trapping and bunching may compensate each other. This idea has been reinvestigated in [Mourenas et al., 2018], where effects of wave modulation were taken into account. Spacecraft observations [Zhang et al., 2019; Zhang et al., 2020b; Foster et al., 2021; Oka et al., 2019; Artemyev et al., 2022] and numerical simulations [Nunn, Omura, 2012; Katoh, Omura, 2016; Demekhov et al., 2017; Tao et al., 2020; Zhang et al., 2021] show that intense whistler-mode waves are mostly propagating in a form of short modulated wave-packets. Typical wave-packets include only few wave periods (Figure 1), that can be an effect of sideband instability of wave generation [Nunn, 1986] or overlapping of several waves with close wave frequencies [Zhang et al., 2020b; Nunn et al., 2021]. Such modulation reduces the efficiency of phase trapping [Tao et al., 2012; Tao et al., 2013], and make a net effect of electron resonant interactions with waves more diffusive [Zhang et al., 2020a; Allanson et al., 2020; Allanson et al., 2021; Gan et al., 2020; An et al., 2022]. Thus, derivation of diffusion rates is the main question for theoretical description of such regime of wave-particle interaction.

In this work, we propose an approach for evaluation of diffusion rates including nonlinear effects for intense, but strongly modulated waves. Firstly, we derive basic model equations for the relativistic electron in the background magnetic field including equations of motion. Secondly, we introduce wave modulations that can fit the experimental data (see Figure 1). Thirdly, we compare the results of computations for the constant amplitude case and for the modulated one to evaluate the reduction of nonlinear effects by modulations.

BASIC CONCEPT AND MAIN EQUATIONS

To propose the approach for evaluation of such diffusion rate, let us illustrate the wave modulation effect on nonlinear wave-particle interactions. We consider electron bouncing in a magnetic trap modelled by curvature free dipole field [Bell, 1984] and their resonant interaction with monochromatic intense whistler-mode wave. Using the wave model from [Vainchtein et al., 2018], we evaluate a set of test particle trajectories resonating once with whistler-mode waves. We start with the Hamiltonian of a relativistic electron (rest mass is m_e and charge is $-e$) bouncing in the magnetic trap and interacting with a field-aligned whistler-mode wave:

$$H = \sqrt{\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 c^2 + m^2 c^4},$$

where $\mathbf{p}$ is a canonical momentum and $\mathbf{A}$ is a vector potential. Potential $\mathbf{A}$ can be derived from the equation $\mathbf{B}=\mathrm{rot}\mathbf{A}$. Magnetic field induction $\mathbf{B}$ consists of two separate terms: $\mathbf{B}=\mathbf{B}_0+\mathbf{B}_{\mathrm{w}}$ where $\mathbf{B}_0$ is the background magnetic field of Earth and $\mathbf{B}_{\mathrm{w}}$ corresponds to the induction of the wave. Strong background magnetic field $\mathbf{B}_0$ makes the coordinate space highly anisotropic. That suggests that the particle will mainly propagate along the magnetic vector $\mathbf{B}_0$. Thus, relations $\partial_z \gg \partial_x, \partial_y$ hold. We define $\mathbf{B}_0$ as a magnetic field of a dipole and can write down $\mathbf{A}$ as

$$A = \left(-\frac{B_w}{k}\cos\phi,\ xB_0 - \frac{B_w}{k}\sin\phi,\ 0\right)$$

where k is the wave number and ϕ is the wave phase. Using this equation for the potential, the Hamiltonian can be written as

$$H = m_e c^2 \times$$
$$\sqrt{(1 + \left(\frac{p_z}{m_e c}\right)^2 + \left(\frac{p_x}{m_e c}\right)^2 + \frac{\Omega_{ce}}{kc}\frac{B_w}{B_0}\cos\phi)^2 + \left(\frac{x\Omega_{ce}}{c} - \frac{\Omega_{ce}^2}{kc}\frac{B_w}{B_0}\sin\phi\right)}$$

where $\Omega_{\mathrm{ce}}=\mathrm{e}B_0/(m_{\mathrm{ec}})$ is the cyclotron frequency of electron. The wavenumber can be determined by the cold plasma approximation: $kc/\Omega_{\mathrm{pe}}=(\Omega_{\mathrm{ce}}/\omega-1)^{-1/2}$, where $\Omega_{\mathrm{pe}} = \sqrt{4\pi n_e e^2 / m_e}$ is the plasma frequency of electrons.

To separate the perturbed and unperturbed parts, it is convenient to introduce another pair of canonical variables (ψ, I_x) where $Ix = (2\pi)^{-1}\oint dx\, p_x$ is the adiabatic invariant of the system

$$x = \frac{c}{\Omega_{ce}}\sqrt{\frac{2I_x\Omega_{ce}}{m_e c^2}}\sin\psi,$$

$$px = m_e c\sqrt{\frac{2I_x\Omega_{ce}}{m_e c^2}}\cos\psi.$$

Substituting these equations into the Hamiltonian, gives

$$H = m_e c^2 \sqrt{(1+\left(\frac{p_z}{m_e c}\right)^2 + \frac{2I_x\Omega_{ce}}{m_e c^2} + 2\sqrt{\frac{2I_x\Omega_{ce}}{m_e c^2}}\frac{\Omega_{ce}}{kc}\frac{B_w}{B_0}\cos(\phi+\psi) + \left(\frac{\Omega_{ce}}{kc} - \frac{B_w}{B_0}\right)^2}$$

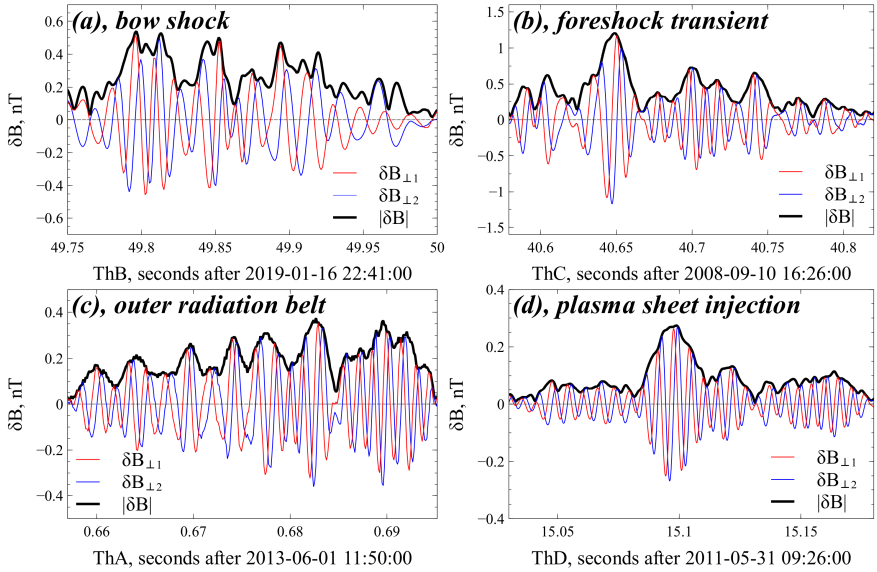

Figure 1. Examples of typical wave-packets of whistler-mode waves captured by THEMIS spacecraft [Angelopoulos, 2008] in Earth bow shock (*a*), foreshock transient (*b*), outer radiation belt (*c*), plasma injection region (*d*). These events are picked up from statistics published in [Artemyev et al., 2022; Shi et al., 2020; Zhang et al., 2018b, a]

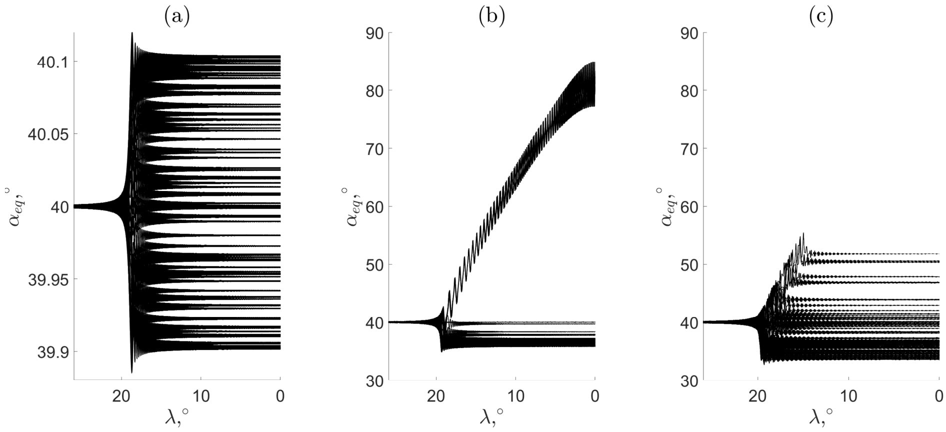

Figure 2. Set of trajectories for low amplitude wave with diffusive scattering (*a*), coherent high-amplitude wave with trapping and bunching (*b*), modulated high-amplitude wave with diffusive-like scattering (*c*). System parameters are: electron energy E_0=100 keV, equatorial pitch-angle α_0=40°, number of particles N=100, wave amplitude B_w=5 pT (*a*), B_w=500 pT (*b*, *c*)

PHASE MODULATION

The Hamiltonian derived in the previous section fully determines the evolution of the system parameters. In a general case, B_w is a function of ϕ. Such modulation of the wave amplitude strongly affects the dynamics of the system as it suppresses the effects of nonlinear interactions. Figure 2 shows the simulation results for various sets of parameters: wave amplitudes and $B_w(\phi)$. The modulations force particles to escape from the trapping, and thus make wave-particle resonant interactions more diffusive.

We define the diffusion coefficient $D_{EE} = \left\langle (\Delta E)^2 \right\rangle$ as a function of the wave intensity and compare the results to the quasi-linear theory. Numerical results presented on Figure 3 suggest that wave modulations effectively reduce the impact of the nonlinear interaction: there is no trapped particles with energy deviations comparable to the initial energy and phase bunching is now greatly balanced by enhanced trapped population. But for the higher wave amplitudes the diffusion process does not obey the quasi-linear theory ($D_{EE}\sim\varepsilon^2$) as its power asymptote goes lower than the theoretical prediction: $D_{EE}\sim\varepsilon^q$ $0<q<2$.

CONCLUSIONS

To conclude, the resonant electron interaction with highly intense whistler-mode waves cannot be described by the quasi-linear theory considering the effect of nonlinear interactions, but the wave intensity modulations

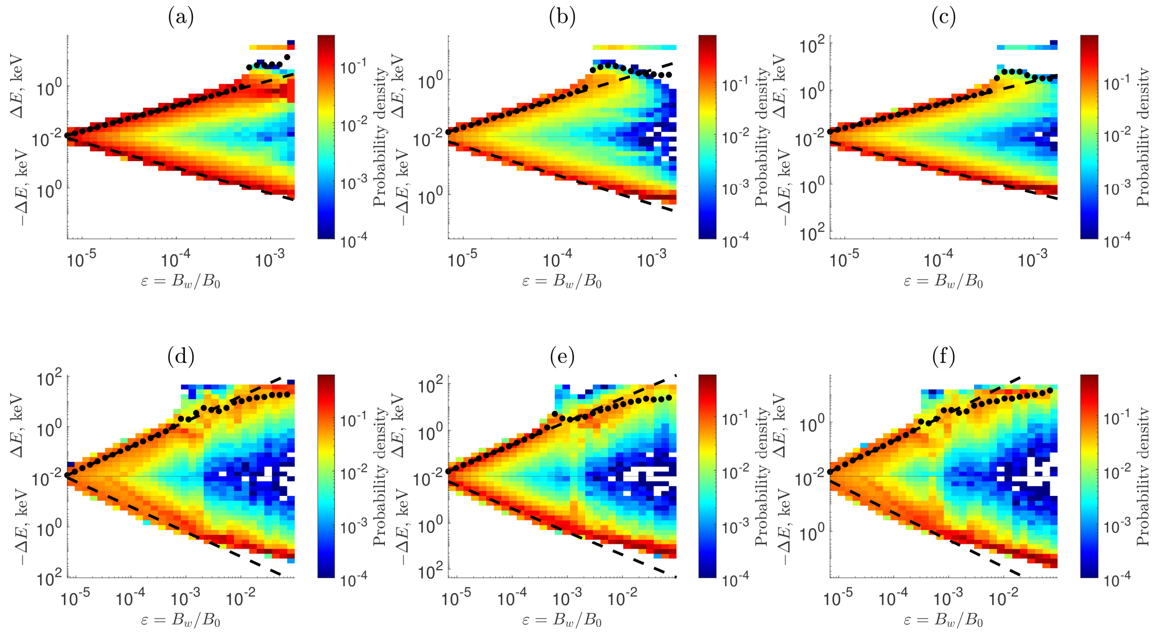

Figure 3. The probability density as a function of the wave amplitude coefficient ε and particle's energy deviation ΔE. The top row corresponds to the system with a constant amplitude and the bottom one – to a modulated intensity. Three sets of initial conditions were taken: E_0=100 keV, α_0=40° (*a*, *d*); E_0=100 keV, α_0=60° (*b*, *e*); E_0=300 keV, α_0=60° (*c*, *f*)

resolves this issue: the modulation of wave amplitude suppresses the trapping process and reduces the net effect of phase bunching. However, the diffusion coefficient D_{EE} derived for such intense modulated waves has a different scaling with the wave amplitude in comparison with one predicted by the quasi-linear theory: $D_{EE} \sim \varepsilon^q$, $0<q<2$. The factor q can be determined from the Hamiltonian equations that were derived in this paper.

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