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ЭФФЕКТ УШИРЕНИЯ РЕЗОНАНСА В ЗАДАЧЕ О РАССЕЯНИИ РЕЛЯТИВИСТСКИХ ЭЛЕКТРОНОВ ИОННО-ЦИКЛОТРОННЫМИ ВОЛНАМИ

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RESONANCE BROADENING EFFECT FOR RELATIVISTIC ELECTRON INTERACTION WITH ELECTROMAGNETIC ION CYCLOTRON WAVES

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Аннотация. Резонансное взаимодействие релятивистских электронов с электромагнитными ионно-циклотронными (ЭМИЦ) волнами является одной из основных причин рассеивания электронов по питч-углу с высыпанием из радиационных поясов Земли в ионосферу. Амплитуда ЭМИЦ-волн, наблюдаемых спутниками, как правило, достаточно велика, чтобы резонансное взаимодействие электронов и волн было нелинейным. Одним из наиболее важных эффектов нелинейного резонансного взаимодействия волна — частица является эффект уширения резонанса. В этой работе мы исследуем данный эффект для реалистичных параметров ЭМИЦ-волн и показываем, что уширение резонанса может значительно расширить диапазон питч-углов электронов, рассеиваемых ЭМИЦ-волнами.

Ключевые слова: радиационные пояса, взаимодействие волна — частица, ЭМИЦ-волны

Abstract. Relativistic electron scattering by electromagnetic ion cyclotron (EMIC) waves is one of the most effective mechanisms for electron flux depletion in the Earth's radiation belts. Resonant electron interaction with EMIC waves is traditionally described by quasi-linear diffusion equations, although spacecraft observations often report EMIC waves with intensities sufficiently large to trigger nonlinear resonant interaction with electrons. In this study we quantify this resonance broadening effect in electron pitch-angle diffusion rates. We show that resonance broadening can significantly increase the pitch-angle range of EMIC-scattered electrons.

Keywords: radiation belts, wave — particle interaction, EMIC waves

INTRODUCTION

Relativistic electron scattering by electromagnetic ion cyclotron (EMIC) waves is one of the most effective mechanisms for electron losses in the Earth's radiation belts [Thorne, Kennel, 1971]. For typical EMIC wave characteristics, the resonant electron energies are above ~1 MeV for field-aligned electrons [Summers, Thorne, 2003], and even higher for higher pitch-angles (the angle between electron velocity and background magnetic field). Such an increase of the resonant energy with pitch-angle implies that EMIC waves cannot scatter the main population of relativistic electrons (at ~1-3 MeV), which are highly anisotropic with maximum fluxes at ~90º pitch-angles. Therefore, the overall contribution of EMIC waves to the net losses of relativistic electrons still requires investigations, although EMIC-scattering rates for resonant (low pitch-angle) electrons can be very large [Thorne, Kennel, 1971].

Several mechanisms have been proposed to extend the energy/pitch-angle range of electrons interacting with EMIC waves, and one of them is the extension of resonant pitch-angle range by the resonance broadening effect [Karimabadi et al., 1992]. The variant of this effect driven by a large wave amplitude has been examined for whistler-mode waves [Cai et al., 2020], but not yet quantified for EMIC waves. As EMIC waves can be very intense with wave amplitudes reaching a few percent of the background magnetic field [Zhang et al., 2016], the finite resonance effect may significantly expand the pitch-angle range of scattered electrons.

Basic equations for wave-particle resonance in inhomogeneous magnetic field

We start our quantification of the resonance broadening effect with the Hamiltonian of a relativistic electron (the rest mass is m_e, charge is $-e$) moving in the electromagnetic field given by the vector potential $A=(A_x, A0+A_y, 0)$

$$H = \sqrt{m_e^2 c^4 + c^2 p_z^2 + (cp_x + eA_x)^2 + (eA_0 + eA_y)^2},$$

Where (z, p_z) are the conjugate pair of the field-aligned coordinate and momentum, (x, p_x) are the conjugate pair of the transverse coordinate and momentum, $A_0(x, z)=xB_0(z/R)$ is the vector potential component of the curvature-free dipole field [Bell, 1984] with the spatial scale $R>>m_e c^2/eB_0$. For the background magnetic field B_0, we use the dipole model $B_0 = B_{eq}\sqrt{1+3\sin^2\lambda}\Big/\cos^6\lambda$, with B_{eq} being the equatorial magnetic field and λ the magnetic latitude. The relation between z and λ is given by the differential equation $dz/d\lambda = R\sqrt{1+3\sin^2\lambda}\cos\lambda$, with R=$R_E L$, L being the L-shell parameter and R_E the Earth radius. Vector potential components for L-mode field-aligned plane waves, $A_{x,y}(z, t)$ are given by the following equation:

$$A_{x,y} = \frac{B_w}{k}\begin{cases} -\cos\phi, \ \partial\phi/\partial z = k \\ \sin\phi, \ \partial\phi/\partial t = -\omega. \end{cases}$$

Wave amplitude B_w is sufficiently small so that $eB_w/(km_e c^2)$<<1. For a fixed wave frequency ω the wave number k is given by the cold plasma dispersion relation [Stix, 1962]

$$\left(\frac{kc}{\omega}\right)^2 = 1 - \frac{\omega_{pe}^2}{\omega(\omega-\Omega_{ce})} - \sum_{i=1}^{3}\frac{\omega_{pi}^2}{\omega(\omega-\Omega_{ci})},$$

where Ω_{ce}=$eB/(m_e c)$, ω_{pe} is the plasma frequency, and indices i=1, 2, 3 are attributed to H^+, He^+, and O^+ ions, respectively.

We will follow the approach proposed in [Shklyar, Matsumoto 2009; Artemyev et al. 2018] and examine this Hamiltonian around the resonance. After introduction of magnetic moment I and electron gyrophase θ and using canonical transformations, we can get the Hamiltonian

$$H \approx m_e c^2\gamma_R - \omega I_R + \frac{1}{2}K^2 P_\psi^2 + A\psi + B\sin\psi,$$

$$\frac{A}{m_e c^2} = -\frac{\partial \ln\Omega_{ce}}{\partial s}\frac{1}{k\gamma_R}\frac{1}{1-(\omega/kc)^2}\times$$

$$\times\left\{\frac{p_R}{m_e c}\frac{\Omega_{ce}}{kc} + \frac{kI_R\Omega_{ce}}{kc} + \frac{\partial\ln k}{\partial\ln\Omega_{ce}}\left(\frac{p_R}{m_e c}\right)^2\right\},$$

where $\psi = \phi - \theta$, $P_\psi = I - I_x$, $I = -I_x$, $I_R = I_R(p, z)$ is taken in the resonance point.

By integrating over trajectory of the scattered electron and averaging over particle ensemble we can get the resonance width for electron energy $m_e c^2\langle\Delta\gamma\rangle$ and equatorial pitch-angle $\langle\Delta\alpha_{eq}\rangle$

$$m_e c^2\langle\Delta\gamma\rangle = \frac{\omega}{\pi}\sqrt{\frac{2B}{K^2}}f\left(\frac{A}{B}\right),$$

$$\langle\Delta\alpha_{eq}\rangle \approx \frac{2\Delta\gamma}{(\gamma^2-1)\sin(2\alpha_{eq})}\frac{\Omega_{ce,\,eq}}{\omega}.$$

Function $f(A/B)$ is determined in [Shklyar, Matsumoto 2009; Artemyev et al. 2018]. Figure 1 shows that $\Delta\alpha_{eq}$ maximizes at low energies, where the effect of the resonance broadening should be most important because of the narrow pitch-angle range of electrons satisfying the exact resonant condition with EMIC wave.

Role of resonance broadening in electron pitch-angle diffusion

After determining the resonance width as a function of wave characteristics, electron energy and pitch-angle, we can include the resonance broadening effect into

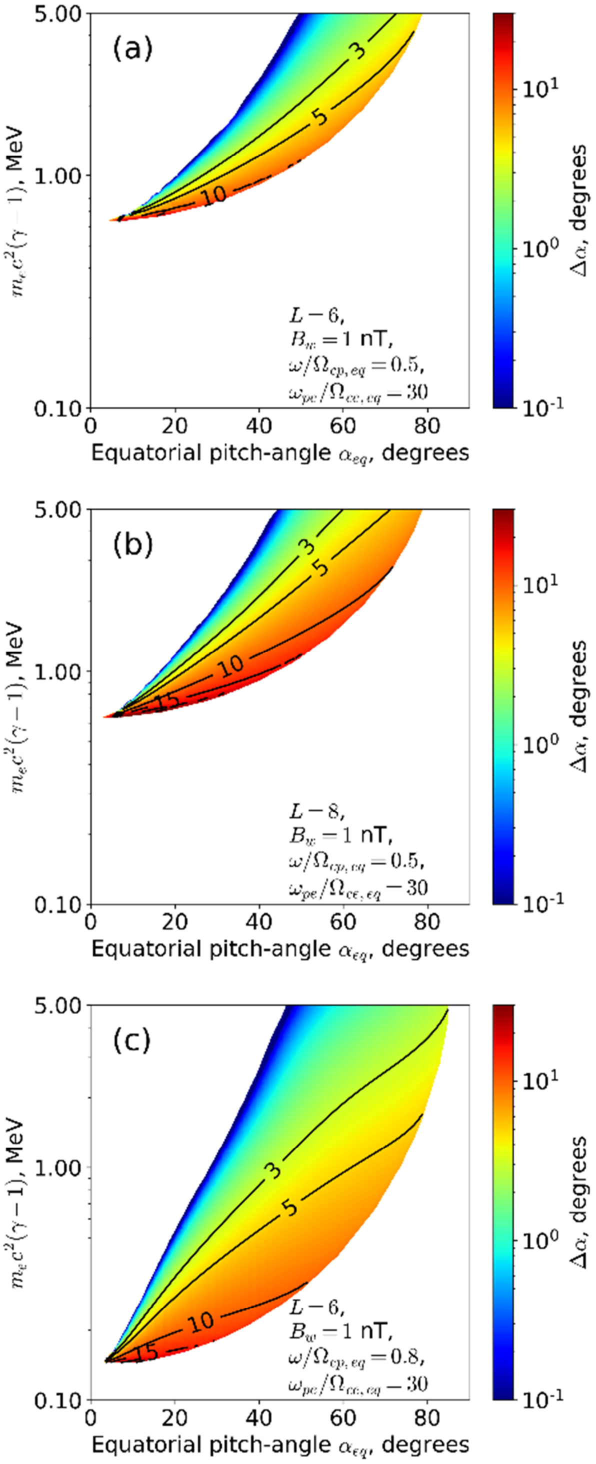

Figure 1. The resonance width in equatorial pitch-angles as a function of (α_{eq}, $m_e c^2(\gamma-1)$) for different background parameters in a case of proton-electron plasma

diffusion rate calculations. We start with the local (at equator) pitch-angle diffusion for field-aligned EMIC waves propagating in a proton-electron plasma with truncated Gaussian spectra [Summers, Thorne, 2003]:

$$D_{\alpha\alpha} = \frac{\Omega_{ce}}{\gamma}\frac{2R}{v\delta x}\int_{x_1}^{x_2}dx\left(g(x-x_r)\frac{x(1-x)}{2-x}e^{-[(x-x_m)/\delta x]^2}\right),$$

where x=ω/Ω_{ci}, Ω_{ci} is the proton gyrofrequency, x_1 and x_2 are lower and upper cut-off frequencies, x_r is the normalized resonance frequency for a given equatorial electron pitch-angle and energy, $R=(B_w/B_0)^2$ is the normalized wave intensity, and $v=\sqrt{\pi}erf(1)\approx 1.49$. In the classical formulation of the quasi-linear diffusion rate function $g=\delta(x-x_r)$ is the Dirac delta function. The finite resonance width can be included into the diffusion rate evaluations by changing g to [Cai et al., 2020; Karimabadi et al., 1992]:

$$g(x)=\begin{cases}(2\Delta x)^{-1}, & |x|<\Delta x\\ 0, & |x|\geq \Delta x\end{cases},$$

$$\Delta x=\frac{k}{\partial k/\partial x}\left[-\frac{\gamma}{\gamma^2-1}\Delta\gamma+\tan\alpha\Delta\alpha\right],$$

the resonance width in frequency is determined from the local resonance condition. Figure 2, *a* shows local (equatorial) diffusion rates with $g=\delta(x-x_r)$ (solid lines) and with resonance width (dashed lines). The resonance broadening effect is most clearly seen as an extension of the pitch-angle range with a finite diffusion rate.

These equations determine the resonance width at the resonant latitude for a given energy and equatorial pitch-angle. Therefore, $\Delta\alpha_{eq}$ and Δx varies with latitudes, and we can incorporate the resonance broadening effect into the bounce-averaged diffusion rate [Lyons et al., 1972] $D_{\alpha_{eq}\alpha_{eq}}\geq\frac{1}{\tau}\int_0^{\lambda_m}D_{\alpha\alpha}\frac{\cos\alpha}{\cos^2\alpha_{eq}}\cos^7\lambda d\lambda$, where we use the dipole magnetic field model with $\sin^2\alpha=\sin^2\alpha_{eq}\sqrt{1+3\sin^2\lambda}/\cos^6\lambda$. The normalized bounce period is $\tau\approx 1.3-0.56\sin\alpha_{eq}$.

Figure 2, *b* shows bounce averaged diffusion rates without (solid curves) and with (dashed curves) the resonance broadening effect included. There is a clear extension of the resonance pitch-angle range to higher pitch-angles due to the finite $\Delta\alpha$. This effect should be most important for low resonant energies, where the exact resonant condition allows only scattering of near loss-cone electrons.

CONCLUSIONS

In this study we quantify the resonance broadening effect for EMIC waves resonating with relativistic electrons. Although perturbation theory requires B_w/B_0 to be small, the resonance width $\sim\sqrt{B_w/B_0}$, may provide an important widening of the resonant pitch-angle ranges for specific plasma populations. An important characteristic controlling the efficiency of the resonance broadening is the ratio of background magnetic field gradient and wave intensity, i.e., ratio A/B. For cases with $|A/B|<1$, the wave intensity is sufficiently high to create a finite resonance width, and the same inequality means that electrons resonate with EMIC waves nonlinearly. Overall, the resonance broadening effect should extend the pitch-angle range of electron scattering for a significant fraction of observed EMIC waves.

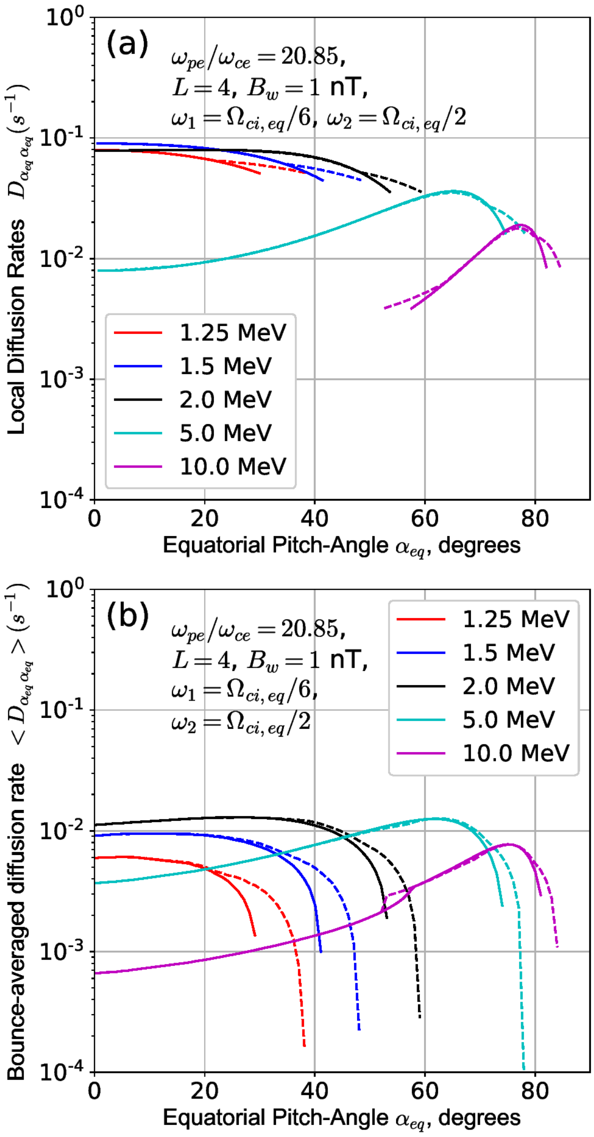

Figure 2. Local (equatorial) (*a*) and bounce-averaged (*b*) diffusion rates without (solid) and with (dashed) the resonance broadening effect included

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