# Alfvén wave parallel electric field in the dipole model of the magnetosphere

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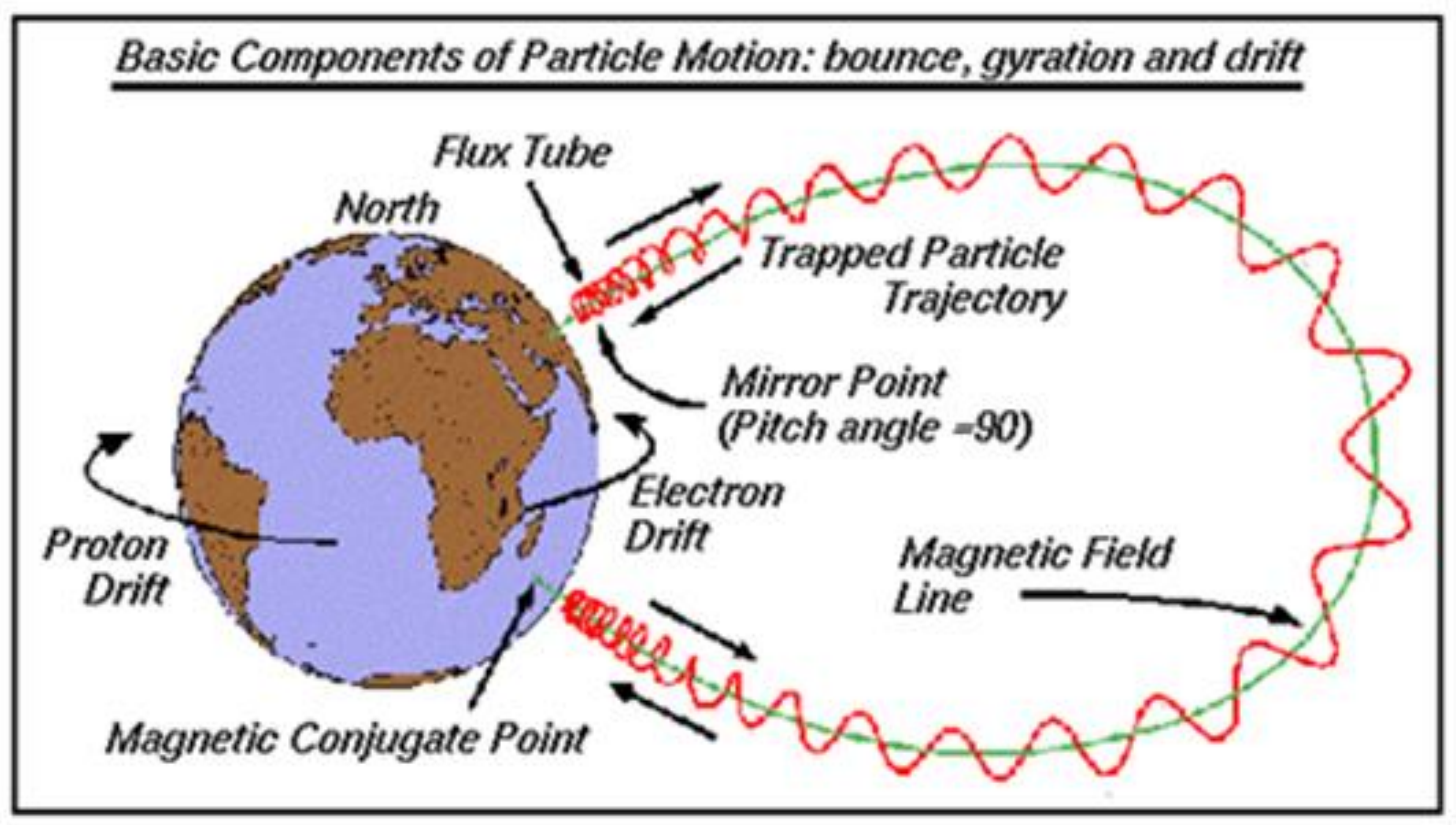
# ULF waves

(T > 0,2 sec)


Basic Components of Particle Motion: bounce, gyration and drift
Flux Tube
North
Trapped Particle
Trajectory
Mirror Point
(Pitch angle =90)
Electron
Drift
Proton
Drift
Magnetic Field
Line
Magnetic Conjugate Point

# Main steps

$$\begin{cases} \hat{L}_A\psi + \hat{L}_C b_\parallel = 0 \\ \hat{L}_C\psi + \hat{L}_M b_\parallel = 0 \\ \hat{L}_\varphi \varphi_\parallel = \hat{L}_\psi \psi + \hat{L}_b b_\parallel \end{cases}$$

[Klimushkin and Mager,2015]

$$\phi_\parallel \ll \psi$$

$$\begin{cases} \psi \\ b_\parallel \approx \dfrac{ck_2}{\omega\sqrt{g_2}}\left(\dfrac{4\pi P'}{\sqrt{g_1}B^2} + \dfrac{3}{2}\dfrac{\beta}{1+\beta}\dfrac{1}{R}\right)\psi \end{cases}$$

$$\vec{E}_\perp = -\nabla_\perp \psi$$

$$\vec{E}_\parallel = -\nabla_\parallel \varphi_\parallel$$

$$\sum_{i,e}\frac{q^2}{m}\int d\mu d\varepsilon\left(\frac{\partial F}{\partial \varepsilon}(\varphi_\parallel + \psi) - J_0^2\frac{\hat{Q}F}{\omega}\psi + \frac{m}{q}J_0\delta k_S\right)\frac{B}{|v_\parallel|} = 0$$

$$\delta k_{S_e} = -\frac{q_e}{m_e}\frac{\hat{Q}F_e}{\omega}\overline{\left(\varphi_\parallel + \frac{\omega_{d_e}}{\omega}\psi + \frac{m_e}{q_e}\mu b_\parallel\right)}$$

$$\overline{(\dots)} = \frac{2}{\tau_b}\int_{-l_0}^{l_0}\frac{(\dots)}{|v_\parallel|}dl_\parallel$$

$$\hat{Q}F = \omega\frac{\partial F}{\partial \varepsilon} + \frac{k_2}{\omega_c\sqrt{g_\perp}}\frac{\partial F}{\partial x^1}$$

$$\tau_b = 2\int_{-l_0}^{l_0}|v_\parallel|^{-1}dl_\parallel$$


## Reference

Chen L., Hasegawa A. Kinetic theory of geomagnetic pulsations. 1. Internal excitations by energetic particles. *J. Geophys. Res.* 1991, vol. 96, pp. 1503–1512. DOI:10.1029/90JA02346.

Klimushkin D. Y., and P. N. Mager The Alfven mde gyrokinetic equation in fint-pressure magnetospheric plasma, *J. Geophys. Res. Space Physics* 2015, vol. 120, pp. 4465–4474. DOI:10.1002/2015A021045.

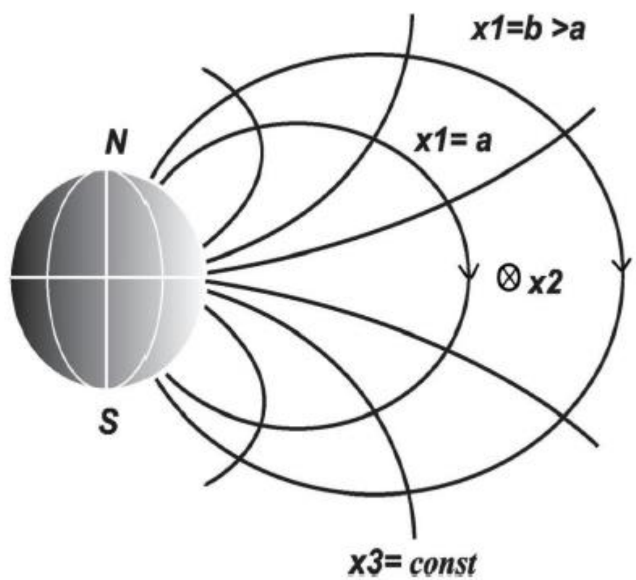
## The distribution function

$$F = \frac{n}{(2\pi V)^{3/2}} exp\left(-\frac{v^2}{2V^2}\right)$$

## Approximation used for the solution

$$k_\perp \rho \ll 1 \qquad \beta \ll 1 \qquad \omega \gg \overline{\omega}_d$$

## Coordinate system



# Analytical solution

$$\phi_{\parallel} - \frac{1}{L}\int_{0}^{l_I} K(l,l')\phi_{\parallel}(l')dl' = \frac{\omega_{d_{e0}}}{\omega}\left[\frac{2\sqrt{g_1}}{R}\psi + \frac{3}{4L}\int_{0}^{l_I}\tilde{K}(l,l')\phi_{\parallel}(l')dl'\right]$$

$$K(l,l') = \theta(l-l')\frac{B(l)}{B_0}\int_{B_0/B_I}^{B_0/B(l)}\frac{d\lambda}{u(\lambda,l)u(\lambda,l')} + \theta(l'-l)\frac{B(l)}{B_0}\int_{B_0/B_I}^{B_0/B(l')}\frac{d\lambda}{u(\lambda,l)u(\lambda,l')}$$

$$\tilde{K}(l,l') = \theta(l-l')\frac{B(l)}{B_0}2\frac{\sqrt{g_1}}{R}\left(\int_{B_0/B_I}^{B_0/B(l)}\frac{\lambda d\lambda}{u(\lambda,l)u(\lambda,l')} - \int_{B_0/B_I}^{B_0/B(l)}\frac{d\lambda}{u(\lambda,l)u(\lambda,l')}\right) +$$
$$\theta(l'-l)\frac{B(l)}{B_0}2\frac{\sqrt{g_1}}{R}\left(\int_{B_0/B_I}^{B_0/B(l')}\frac{\lambda d\lambda}{u(\lambda,l)u(\lambda,l')} - \int_{B_0/B_I}^{B_0/B(l')}\frac{d\lambda}{u(\lambda,l)u(\lambda,l')}\right)$$

$$u(\lambda,l) = \sqrt{1-\lambda B(l)/B_0}$$

# Numerical solution

Poloidal wave case $k_1 = 0$

$\vec{E}_a = 10mV/m$

$k_2 = 50\ (azimutal\ wave\ number)$

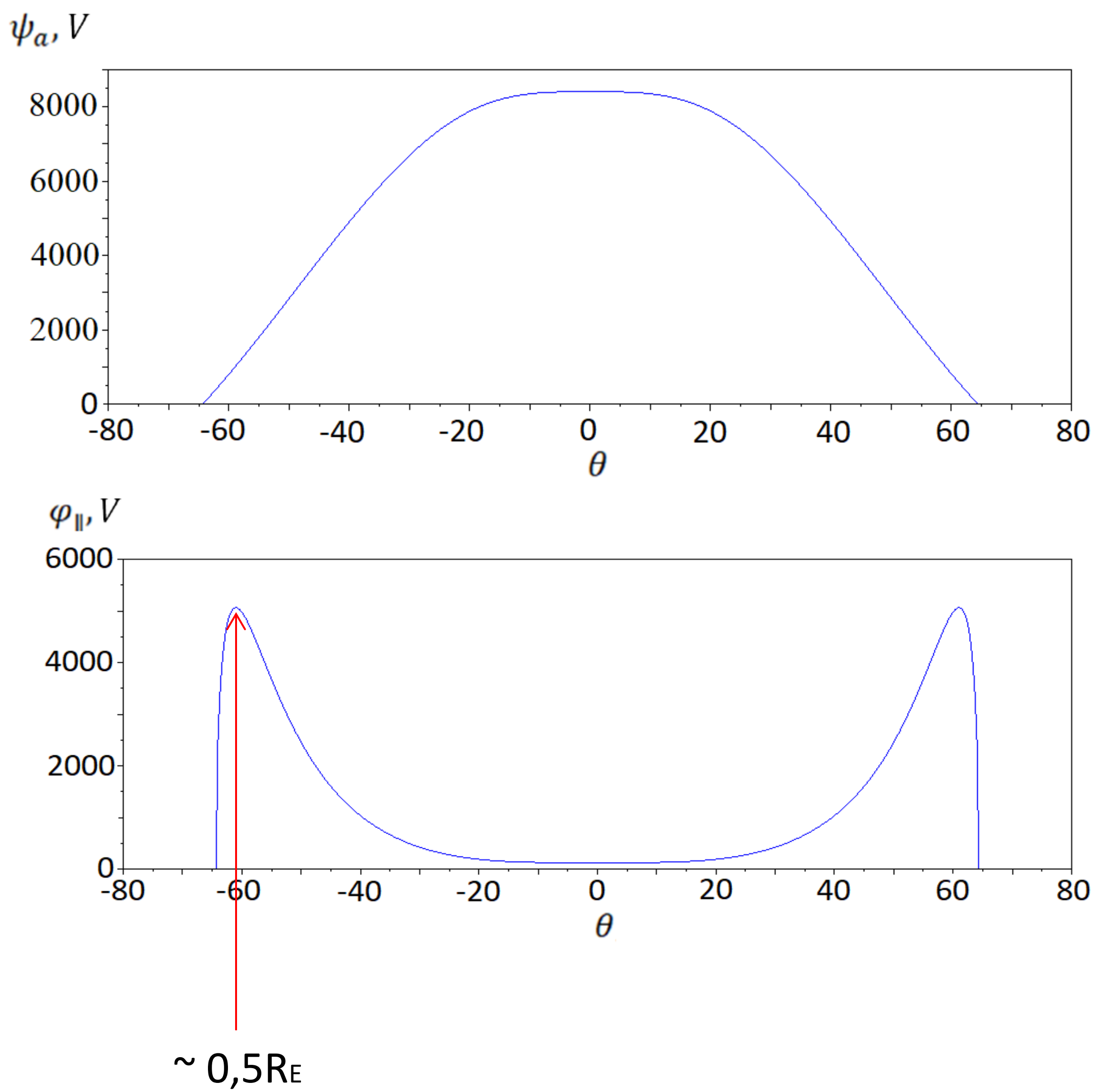
$V_A = 1200km/s$

$L = 6.6R_E$

$R_I = R_E + 1500km$

$\omega = 11.8mHz$

$T_e = 1keV$



Thank you for your attention!